

Roll no.: 2024006193

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Degree: B.Tech
End Semester Examination, April-May, 2025

Semester: II

Course Title: Mathematics II
Maximum Marks: 50

Course Code: FCMT0201
Time: 3 hours.

Note: Attempt all the five questions. Missing data/information (if any), may be suitably assumed and mentioned in the answer.

Q. No.	Question	Marks	CO
Q1	Attempt any 2 parts of the followings:	5	CO
1a	Find the general solution of the differential equation: $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$	5	CO
1b	Find the complete solution of the differential equation: $(D^2 - 4D + 4)y = 8x^2e^{2x} \sin 2x.$	5	CO
1c	Using Frobenius method find the series solution about the point $x = 0$ of the differential equation: $x(1-x)y'' - (1+3x)y' - y = 0.$	5	CO
Q2	Attempt any 2 parts of the followings:		
2a	Using row operations, solve completely the following homogeneous system of equations: $4x + 3y - z = 0, 3x + 4y + z = 0, x - y - 2z = 0, 5x + y - 4z = 0.$	5	CO
2b	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and use it to find the matrix $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$	5	CO
2c	Find all the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$	5	
Q3	Attempt any 2 parts of the followings:		
3a	Using Newton-Raphson method, find an approximate root of the equation $xe^x = 3x$ correct to four decimal places, lying between 0 and 1.	5	
3b	Using Simpson's 1/3 rule, evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin 2x} dx$ (correct upto 4 decimal places) by dividing the interval $[0, 1]$ into six equal parts. Compare the results with Simpson's 3/8 rule and Trapezoidal rule.	5	

3c Consider $\frac{dy}{dx} = x + y^2$, where $y(0) = 1$. Find an approximate value of y at $x = 0.1$ correct to four decimal places, by fourth order Runge-Kutta method. 5

Q4 Attempt any 2 parts of the followings:

4a Determine the analytic function whose real part is given by $\frac{y}{x^2+y^2}$. 5

4b Using Cauchy's integral formula, evaluate the integral 5

$$\int_C \frac{e^{-z}}{(z+i)(z-2)^2} dz$$

where C is the circle given by $\{z : |z| = 3\}$.

4c Determine the poles of the function $f(z) = \frac{1}{(z-1)^2(z-2)}$ and residue at each pole using series expansion. 5

Q5 Attempt any 2 parts of the followings:

5a An insurance company found that 0.01% of the population is involved in certain type of accident in each year. If its 1000 policy holders were randomly selected from the population, then what is the probability that not more than two of its clients are involved in such an accident next year? (Given that $e^{-0.1} = 0.9048$). 5

5b In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (i) less than 1950 hours (ii) more than 2150 hours and (iii) more than 1920 hours but less than 2160 hours. (Use $\phi(1) = 0.3413$, $\phi(1.33) = 0.4082$, $\phi(1.83) = 0.4664$, $\phi(2) = 0.4772$). 5

5c Calculate the correlation coefficient between the variables X and Y having the following data: 5

X	10	6	9	10	12	13	11	9
Y	9	4	6	9	11	13	8	4

Also, find the line of regression of the variable X on Y .